International Journal of Engineering Research (ISSN : 2319-6890)
Volume No.2, Issue No.2, pp : 113-118 01 April 2013

Effects of Radiation and free Convection Currents on Unsteady Couette Flow between two Vertical Parallel Plates with Constant Heat flux and Heat Source Through Porous Medium

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ABSTRACT

The present study the free convection in unsteady Couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation with heat source in the presence of uniform magnetic field is presented. The flow is induced by means of Couette motion and free convection currents occurring as a result of application of constant heat flux on the wall with a uniform vertical motion in its own plane while constant temperature on the stationary wall. The fluid considered here is a gray, absorbing-emitting but non-scattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the analysis. The dimensionless governing partial differential equations are solved by using regular perturbation technique. The results for the velocity, temperature and the skin-friction are shown graphically. The effects of different parameters are discussed.

Keywords: Couette flow, Natural convection, Vertical plate, Constant heat flux, Radiation and heat source.

1 INTRODUCTION

Unsteady free convection flows in a porous medium have received much attention in recent time due to its wide applications in geothermal and oil reservoir engineering as well as other geophysical and astrophysical studies. Moreover, considerable interest has been shown in radiation interaction with convection for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing-emitting fluids. The unsteady fluid flow past a moving plate in the presence of free convection and radiation were studied by Grief et al. [8] and Makinde [13]. From the previous literature survey about unsteady fluid flow, we observe that little papers were done in porous medium. The effect of radiation on MHD flow and heat transfer must be considered when high temperatures are reached. El-Hakiem [7] studied the unsteady MHD oscillatory flow on free convection-radiation through a porous medium with a vertical infinite surface that absorbs the fluid with a constant velocity. Cookey et al. [6] researched the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past on infinite heated vertical plate in a porous medium with time-dependent suction.

The fluid flow between parallel plates by means of Couette motion is a classical fluid mechanics problem that has applications in magnetohydrodynamic (MHD) power generators and pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil, and also in many material processing applications such as extrusion, metal forming, continuous casting, wire and glass fiber drawing, etc. This problem has received considerable attention in the case of horizontal parallel plates, Attia, Choi et.al Kumar et. al and Soundalgekar [2, 4, 12, 16] than vertical parallel plates.

In many engineering applications such as cooling of electronic equipments, design of passive solar systems for energy conversion, cooling of nuclear reactors, design of heat exchangers, chemical devices and process equipment, geothermal systems, and others. However, very few papers deal with free convection in Couette motion between vertical parallel plates. Jha [11] Fully-developed laminar free convection Couette flow between two vertical parallel plates with transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by constant suction through the plate in uniform motion has been analyzed by Jain and Gupta [10]. In their study, the moving wall is thermally insulated and the wall at rest is kept at a uniform temperature. Narahari [14] Effects of thermal radiation and free convection currents on the unsteady couette flow between two vertical parallel plates with constant heat flux at one boundary.

The study of heat generation/absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Hosssain et al. [9] studied the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation / absorption. Alam et al. [1] studied the problem of free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of a magnetic field and heat generation. Chamkha [3] investigated unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption.

The aim of the paper is to provide an exact analysis of unsteady free convection in Couette motion between two vertical parallel plates in the presence of thermal radiation in the presence of uniform magnetic field where the moving plate is subject to constant heat flux and the plate at rest is isothermal. These solutions are useful to gain a deeper knowledge of the underlying physical processes and it provides the possibility to get a benchmark for numerical solvers with reference to basic flow configurations.

2 MATHEMATICAL ANALYSIS

Consider the unsteady free-convective Couette flow of an incompressible viscous radiating fluid between two infinite vertical parallel plates in the presence of uniform magnetic field separated by a distance h. The $x'-$ axis is taken along one of the plates in the vertically upward direction and the $y'-$axis is taken normal to the plate. Initially, at time $t' \le 0$, the two plates and the fluid are assumed to be at the same temperature T_h' and stationary. At time $t' > 0$, the plate at y' = 0 starts moving in its own plane with an impulsive velocity U and is heated by supplying heat at constant rate whereas the plate at $y' = h$ is stationary and maintained at a constant temperature T_h'. It is also assumed that the radiative heat flux in the $x'-$ direction is negligible as compared to that in the $y'-$ direction. As the plates are infinite in length, the velocity and temperature fields are functions of y' and t' only. Then under the usual Boussinesq's approximation, the flow of a radiating fluid is shown to be governed by the following system of equations:

$$\frac{\partial u'}{\partial t'} = g\left(T' - T_h'\right) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^{\ 2}}{\rho} u' - \frac{\nu}{K} u'$$

(1)

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} - Q_0 \left(T' - T_h'\right)$$

(2)

With the following boundary conditions

$$t' \le 0: \quad u' = 0, T' = T_h' \qquad for \quad 0 \le y' \le h$$

$$t' > 0: \quad u' = U, \frac{\partial T'}{\partial y'} = \frac{q}{k} \qquad at \quad y' = 0$$

$$u' = 0, T' = T_h' \quad at \qquad y' = h$$

(3)

where g is the acceleration due to gravity, β the volumetric co-efficient of thermal expansion, ν the kinematic viscosity, ρ the density, k the thermal conductivity, C_p the specific heat at constant pressure, q the constant heat flux, q_r the radiative heat flux in $y'-$ direction, T' the fluid temperature, and u' is the fluid velocity.

The radiative heat flux term is simplified by making use of the Rosseland approximation Siegel and Howell [15] as

$$q_r = \frac{4\sigma}{3k^*} \frac{\partial T'^4}{\partial y'}$$

(4)

Where σ is the Stefan - Boltzmann constant and k^* is the mean absorption coefficient. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature, Then the Taylor series for T'^4 about T_h', after neglecting higher order terms, is given by

$$T'^4 \cong 4T_h'^3 T' - 3T_h'^4$$

(5)

It is emphasized here that equation (5) is widely used in computational fluid dynamics involving radiation absorption problems Chung [5] in expressing the term T'^4 as a linear function.

In view of Equations (4) and (5), Equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma}{3k^*} \frac{\partial T_h'^3}{\partial y'} \frac{\partial^2 T'}{\partial y'^2}$$

(6)

In order to solve the governing equations in dimensionless form, we introduce the following non-dimensional quantities:

$$y=\frac{y'}{h},\ t=\frac{t'\nu}{h^2},\ u=\frac{u'}{U},\quad \theta=\frac{T'-T_h'}{\left(hq/k\right)},\ Gr=\frac{g\beta h^3 q}{Uk\nu}$$

$$M=\frac{\sigma B_0{}^2 h^2}{\rho\nu},K=\frac{h^2}{k}\phi=\frac{Q_0 h^2}{\rho C_p\nu},\mathrm{Pr}=\frac{\mu C_p}{k},R=\frac{kk^*}{4\sigma T_h'^3}$$

(7)

where Gr is the thermal Grashof number, Pr the Prandtl number, R the radiation parameter, ϕ heat source parameter, t the dimensionless time, u the dimensionless velocity, y the dimensionless coordinate axis normal to the plate, μ the coefficient of viscosity and θ is the dimensionless temperature. Then in view of Equations (7), Equations (1), (6) and (3) reduces to the following non-dimensional form of equations:

$$\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial t}-Mu-\frac{1}{K}u+Gr\theta$$

(8)

$$\left(3R+4\right)\frac{\partial^2\theta}{\partial y^2}-3R\,\mathrm{Pr}\frac{\partial\theta}{\partial t}-3R\,\mathrm{Pr}\,\phi\theta$$

(9)

The initial and boundary conditions are

$$t\le 0:\quad u=0,\theta=0\qquad for\quad 0\le y\le 1$$

$$t>0:\quad u=1,\frac{\partial\theta}{\partial y}=-1\qquad at\ \ y=0$$

$$u=0,\theta=0\quad at\qquad y=1$$

(10)

3 SOLUTION OF THE PROBLEM

Equation (8) – (9) are coupled, non – linear partial differential equations and these cannot be solved in closed – form using the initial and boundary conditions (10). However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the fluid in the neighbourhood of the plate as

$$f=f_0\left(y\right)+\varepsilon e^{nt}f_1\left(y\right)\ +0\left(\varepsilon^2\right)\tag{11}$$

Substituting (11) in Equation (8) – (9) and equating the harmonic and non – harmonic terms, and neglecting the higher order terms of $0\left(\varepsilon^2\right)$, we obtain

$$u_0''-\beta_1 u_0=-Gr\,\theta_0\quad (12);\quad u_1''-\beta_4 u_1=-\beta_5\theta_1$$

(13)

$$\theta_0''-\beta_9\,\theta_0=0\qquad (14);\quad \theta_1''-\beta_{10}\,\theta_1=0\tag{15}$$

The corresponding boundary conditions can be written as

$$u_0=1,u_1=0,\ \frac{\partial\theta_0}{\partial y}=-1,\frac{\partial\theta_1}{\partial y}=0,\quad at\ \ y=0$$
$$u_0\to 0,u_1\to 0,\ \theta_0\to 0,\theta_1\to 0,\quad as\ \ y\to 1\tag{16}$$

The solutions of Equations (12) - (15) under the initial and boundary conditions (16) by perturbation technique is given by

$$u\left(y,t\right)=Z_1e^{m_2y}+Z_2e^{m_1y}+Z_3e^{m_6y}+Z_4e^{m_2y}$$

$$\theta\left(y,t\right)=D_1e^{m_2y}+D_2e^{m_1y}$$

APPENDIX

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4 RESULTS AND DISCUSSION

An exact solution to the problem of natural convection in unsteady Couette flow between two long vertical parallel plates in the presence of constant heat flux and thermal radiation have been presented in the preceding section. In order to get the physical insight into the problem, the numerical values of the temperature field, the velocity field, the skin-friction, the Nusselt number, the volume flow rate and the vertical heat flux are computed for different values of the system parameters such as Radiation parameter (R), Grashof number $\left(Gr\right)$, Prandtl number (Pr) and heat source parameter$\left(\phi\right)$. Figure (1) presents the velocity profiles for both air and water $\left(\mathrm{Pr}=7.0\right)$ in the case of pure convection $\left(R\to 100\right)$ for different values of Gr. It is seen that the velocity of air and water increases with increasing Gr. At a smaller Gr the velocity distribution is monotonic, but at a higher time it passes through a maximum near the moving plate when the buoyancy effect partly suppresses the inertial effects of the plate velocity. Moreover, the velocity of air is greater than the velocity of water. Physically this is possible because fluids with high Prandtl number have greater viscosity, which makes the fluid thick and hence move slowly. Figure (2) presents the velocity profiles of air for different values of K. It is observed that the velocity increases with increasing K. Physically this is possible because as the Grashof number or time increases, the contribution from the buoyancy force near the moving hot plate become more significant and hence a small rise in the fluid velocity near the plate is observed. Figure (3) illustrate the influences of M

(Magnetic parameter) on velocity profiles respectively. It is found that the velocity decreases with increase of magnetic parameter M. It is also found that the velocity decreases away from the plate and becomes minimum and finally takes asymptotic value. Finally here we also see that point of separation takes place for different values of magnetic parameter. Figure (4) present typical profiles for the velocity, for various values of a heat source (ϕ) respectively. As shown, the velocity decreasing with increasing ϕ. In the event that the strength of the presence of a heat source (ϕ) effect causes a reduction in the thermal state of the fluid, thus producing lower thermal boundary layers. The effect of the Prandtl number on the velocity shown in figure (5). As the Prandtl number increases, the velocity decreases. Further, figure (6) it is observed that the fluid velocity decreases with increasing value of *R*. This result may be explained by the fact that an increase in the radiation parameter $R = \left(\frac{kk^*}{4\sigma T_h'}\right)$ for fixed *k* and T_h' means an increase in the Rosseland mean absorption coefficient k^*. When radiation is present, the momentum boundary layer was found to be thicken, which is in agreement with the observation made earlier with regard to the temperature variations of air. Figure (7) present typical profiles for the temperature for various values of a heat source (ϕ) respectively. As shown, the temperature decreasing with increasing. Figure (8) shows that the temperature profiles for different values of Prandtl number (Pr). It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that increasing values of Prandtl number equivalent to increase the thermal conductivities and therefore heat is able to diffuse away from the heated plate more rapidly. Hence in the case of increasing Prandtl numbers, the boundary layer is thinner and the heat transfer is reduced. Boundary layer suction is the technique in which air pumps is used to extract the boundary layer at the wing. Further, increment of suction parameter decreases the fluid temperature. Figure (9) shows the temperature profiles for different values of the Radiation parameter *R*, it is noticed that an increase in the radiation parameter results decrease in the temperature with in boundary layer, as well as decreased the thickness of the temperature boundary layers. Figure (10) presents the skin-friction variation with *M* in the pure convection case for different values of Gr at the moving plate. It is observed that the skin-friction increases with increasing Gr.

5 CONCLUSIONS

The temperature of the fluid increases with increasing time whereas it decreases due to an increase in the value of radiation parameter. In the case of pure convection (i.e. in the absence of radiation), the velocity of the fluid increases with increasing Grashof number, but falls owing to an increase in the Prandtl number. The velocity of the fluid increases with increasing Grashof number and time but it decreases owing to an increase in the value of the radiation parameter. The skin-friction at the moving plate increases with increasing values of Grashof number and time for air flows.

We may conclude therefore, that the interaction between the radiation, buoyancy forces and the applied shear induced by a uniform vertical motion of the hot wall can affect the configuration of the flow field significantly.

REFERENCES

[1] Alam M S, Rahman M M and Sattar M A: MHD Free convection heat and mass transfer flow past an inclined surface with heat generation, *Thamasat. Int. J. Sci. Tech*. 11 (4) (2006), pp. 1 – 8.

[2] Attia H A: The Effect of Variable Properties on the Unsteady Couette Flow with Heat Transfer Considering the Hall Effect, *Communications in Nonlinear Science and Numerical Simulation*,13, (2008), pp. 1596- 1604.

[3] Chamkha A J: Unsteady MHD convective heat and mass transfer past a semi- infinite vertical permeable moving plate with heat absorption, *Int. J. Eng. Sci.* 24 (2004), pp. 217 – 230.

[4] Choi C K, Chung T J and Kim M C: Buoyancy Effects in Plane Couette Flow Heated Uniformly From Below with Constant Heat Flux, *International Journal of Heat and Mass Transfer*, Vol. 47, (2004), pp. 2629-2636.

[5] Chung T J, Computational Fluid Dynamics, *Cambridge University Press*, 2002.

[6] Cookey C I, Ogulu A and Omubo-Pepple V M: Influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction, *Int. J. Heat Mass Transfer*, 46, (2003), pp. 2305 – 2311.

[7] El-Hakiem M A: MHD oscillatory flow on free-convection radiation through a porous medium with constant velocity, *J. Magn. And Magnetic Mater.* 220 (2, 3), (2000), pp. 271 – 276.

[8] Grief R, Habib I S and Lin J C: Laminar convection of radiating gas in a vertical channel, *J. Fluid Mech*. 46, (1971), pp. 513 – 520.

[9] Hossain M A, Molla M M and Yaa L S: Natural convection flow along a vertical wavy surface temperature in the presence of heat generation/absorption, *Int. J. Thermal Science*, 43, (2004), pp. 157 – 163.

[10] Jain N C and Gupta P: Three Dimensional Free Convection Couette Flow with Transpiration Cooling, *Journal of Zhejiang University SCIENCE A*, 3, (2006), pp. 340-346.

[11] Jha B K: Natural Convection in Unsteady MHD Couette Flow, *Heat and Mass Transfer*, 37, (2001), pp. 329-331.

[12] Kumar J, Lakshmana Rao C and Massoudi M: Couette Flow of Granular Materials, *International Journal of Non-Linear Mechanics*, 38, (2003), pp. 11-20.

[13] Makinde O D: Free-convection flow with thermal radiation and mass transfer past a moving vertical porous plate, *Int. Comm., Heat Mass Transfer*, 32, (2005), pp. 1411 – 1419.

[14] Narahari M: Effects of Thermal Radiation and Free Convection Currents on the Unsteady Couette Flow Between Two Vertical Parallel Plates with Constant Heat Flux at one Boundary, Wseas transactions on heat and mass transfer, 1 (5), (2010)

[15] Siegel R and Howell J R, *Thermal Radiation Heat Transfer*, 4th Edition, *Taylor & Francis*, (2002).

[16] Soundalgekar V M: Hall Effects in MHD Couette Flow with Heat Transfer, *IEEE Transactions on Plasma Science*, Vol. PS-14, (5), (1986), pp. 579-583.

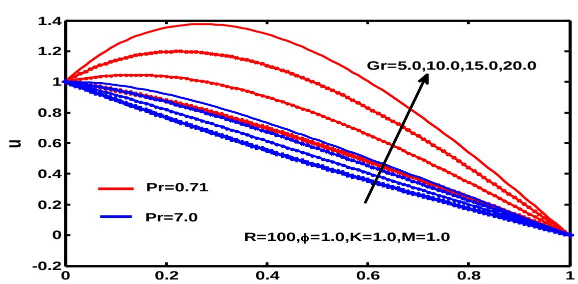

Figure 1: Velocity profiles for different values of Gr (Pure convection case)

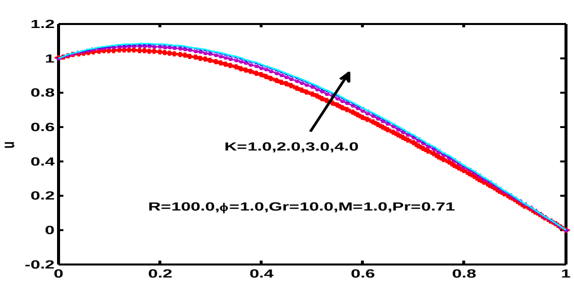

Figure 2: Velocity profiles for different values of K

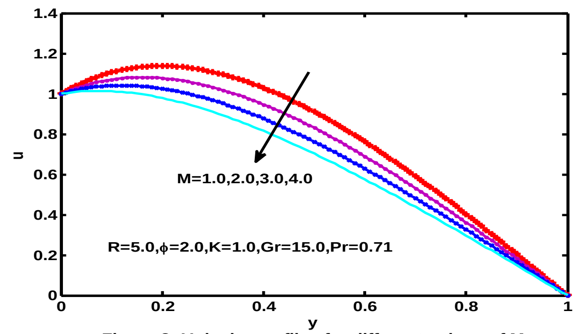

Figure 3: Velocity profiles for different values of M

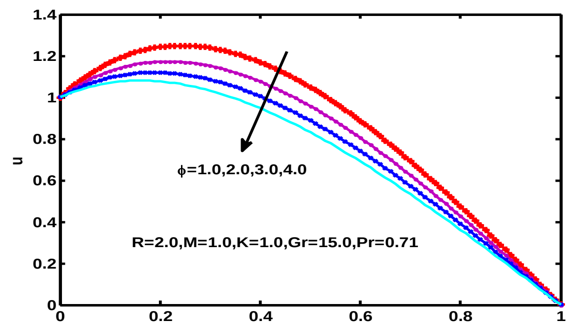

Figure 4: Velocity profiles for different values of ϕ

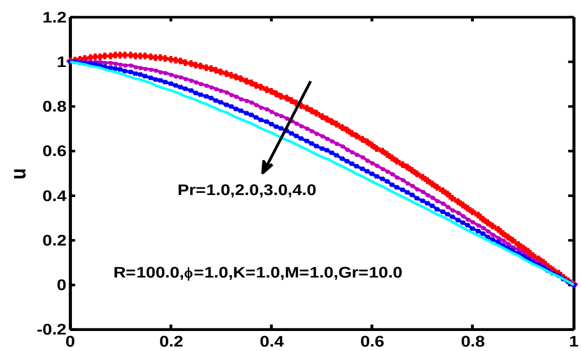

Figure 5: Velocity profiles for different values of Pr

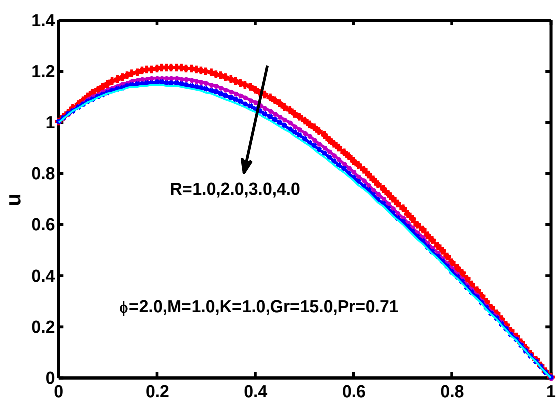

Figure 6: Velocity profiles for different values of R

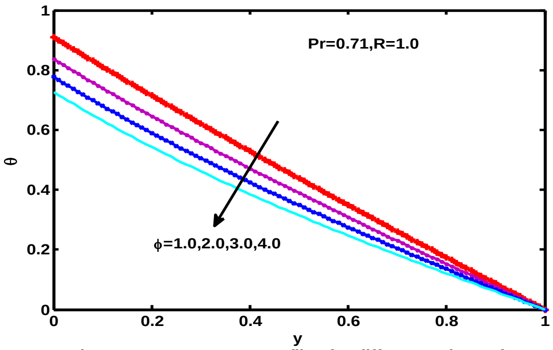

Figure 7.: Temperature profiles for different values of Pr

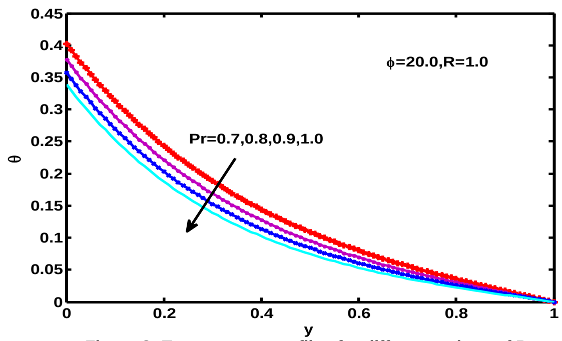

Figure 8: Temperature profiles for different values of Pr

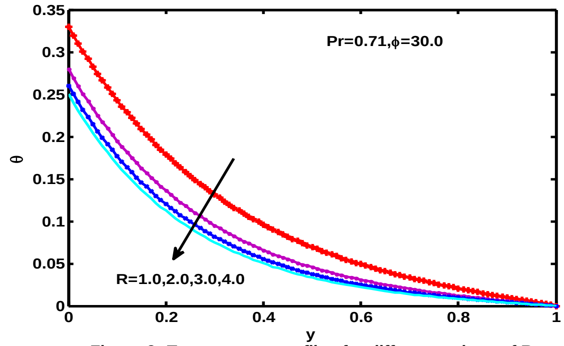

Figure 9: Temeprature profiles for different values of R

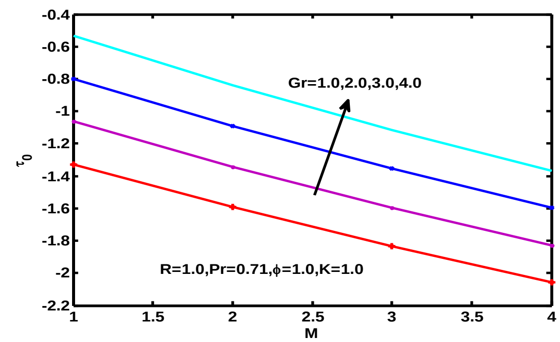

Figure 10: Skin friction for different values of Gr versus M